Loudspeaker Directivity Improvement Using Low Pass and All Pass Filters
**Example Loudspeaker System**

For models and graphs

LF drivers: 250 mm (10 inches)  
HF horn dimension: 64 mm (2.5 inches)  
d = 170 mm (6.75 inches), r = 20 m (65.5 feet)

Omni-directional point sources are used in the models
Vertical Directivity Map – Two LF Drivers

Polar graph – 1/3 octave
500 Hz and 1,000 Hz

Frequency response
at 60° off-axis
Lowest frequency null at a given angle

\[
f_x = \frac{1}{2} c \star \left( \frac{(r \cos \theta)^2 + (d + r \sin \theta)^2}{\left( (r \cos \theta)^2 + (d - r \sin \theta)^2 \right)^{1/2}}, \frac{(r \cos \theta)^2 + (d - r \sin \theta)^2}{\left( (r \cos \theta)^2 + (d + r \sin \theta)^2 \right)^{1/2}} \right)^{-1}
\]

If this frequency is within the pass band of the LF drivers (i.e. below the crossover frequency), the output from one of the drivers must be attenuated in order to minimize the comb filtering.

A low pass filter may be used to accomplish this.
Example Loudspeaker System

Crossover: 1.2 kHz
4th order Linkwitz-Riley

Frequency response on-axis
Example Loudspeaker System

Both LF drivers active

Crossover: 1.2 kHz
4th order Linkwitz-Riley

One LF driver active
(one muted)
Determining Low Pass Filter Parameters

\[ f_\phi = \frac{\phi}{360} \ast \frac{c}{|r_1 - r_2|} \]

\[ r_1 = \left[ (r \cos \theta)^2 + (d + r \sin \theta)^2 \right]^{\frac{1}{2}} \]

\[ r_2 = \left[ (r \cos \theta)^2 + (d - r \sin \theta)^2 \right]^{\frac{1}{2}} \]

Used to determine the frequency of any arbitrary phase difference between the two LF drivers for a given driver spacing, off-axis angle and observation distance.

Below \( f_{120^\circ} \) there is only constructive interference
Above \( f_{120^\circ} \) there is both constructive & destructive interference (comb filtering)

At \( f_{180^\circ} \) there is maximum cancellation
Determining Low Pass Filter Parameters

\[
\frac{P_1}{P_2} = \frac{1 + P_{R \text{off-axis}}}{1 - P_{R \text{off-axis}}}
\]

Required pressure ratio of the individual LF drivers given the desired off-axis pressure ratio

\[
P_{R \text{off-axis}} = \frac{P_{T180^\circ}}{P_{T0^\circ}}
\]

Ratio of the off-axis pressure where there is maximum cancellation to the on-axis pressure

Performance Criterion

Level of \( P_{R \text{off-axis}} \) should be no greater than -1 dB

\[
P_{R \text{off-axis}} = 10^{(-1/20)} = 0.8913
\]

\[
P_1/P_2 = 17.39 \quad 20\log(17.39) = 24.8 \text{ dB}
\]

The level difference between the two LF drivers at \( f_{180^\circ} \) should be 24.8 dB

A 4\(^{\text{th}}\) order low pass filter with a cut-off frequency of approximately \( f_{90^\circ} \), one octave below \( f_{180^\circ} \), may be used to yield this attenuation
Example Loudspeaker System

Crossover: 1.2 kHz
4\textsuperscript{th} order Linkwitz-Riley

LPF on one LF driver: 250 Hz
4\textsuperscript{th} order Linkwitz-Riley
Example Loudspeaker System

Crossover: 1.2 kHz
4th order Linkwitz-Riley

LPF on one LF driver: 250 Hz
4th order Linkwitz-Riley
Phase Compensation

$\varphi_1$ is an All Pass filter with the same phase response as $G_{L1}$
(Phase shift but with flat magnitude response)
Example Loudspeaker System

Polar graph – 1/3 octave
500 Hz and 315 Hz (with & w/o filtering)

Crossover: 1.2 kHz
4th order Linkwitz-Riley

LPF on one LF driver: 250 Hz
4th order Linkwitz-Riley

APF on other LF driver: 250 Hz
2nd order Butterworth

Frequency response on-axis

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Example Loudspeaker System

Crossover: 1.2 kHz
4th order Linkwitz-Riley

LPF on one LF driver: 250 Hz
4th order Linkwitz-Riley

APF on other LF driver: 250 Hz
2nd order Butterworth

Crossover: 1.2 kHz
4th order Linkwitz-Riley
Example Loudspeaker System

Polar graph
1/3 octave, 1.25 kHz
(crossover region)
Example Loudspeaker System

Detrimental Effects

Phase response

Crossover and Low Pass / All Pass Filters

Crossover Only

Impulse response
Real Loudspeaker System

Desirable to have the directivity response of the LF pass band match that of the HF pass band.

Careful spacing of the LF drivers and choice of the LP/AP filter frequency can help extend directivity control below the crossover frequency.

\[
\frac{P_1}{P_2} = \frac{1 + P_{Roff-axis}}{1 - P_{Roff-axis}}
\]

\[
P_{Roff-axis} = \frac{P_{T180°}}{P_{T0°}} = 0.5 \quad (-6\, \text{dB})
\]

\[
P1/P2 = 3.01 \quad 20\times\log(3.01) = 9.6\, \text{dB}
\]

The level difference between the two LF drivers at $f_{180°}$ should be 9.6 dB
Real Loudspeaker System

A 4th order Linkwitz-Riley low pass filter has an attenuation of 9.6 dB at a frequency approximately 1.19 times its cut-off frequency, $f_c$.

Reduce the LP/AP filters $f_c$ by a reciprocal amount, 0.84. The original $f_c$ will now have an attenuation of 9.6 dB.
Real Loudspeaker System

Vertical Directivity Map

HF horn only

Beamwidth

(-6 dB angles)

60°
ranges from

50° - 70°

For \( \theta = 30° \) and \( \varphi = 180° \)

\[ f_{180°} = 1,000 \text{ Hz} \quad f_c = 840 \text{ Hz} \]

Lowered 1/6 octave \( f_c = 750 \text{ Hz} \)

\[ f_\varphi = \frac{\varphi}{360} \cdot \frac{c}{|r_1 - r_2|} \]

\[ r_1 = \left[ (r \cos \theta)^2 + (d + r \sin \theta)^2 \right]^{\frac{1}{2}} \]

\[ r_2 = \left[ (r \cos \theta)^2 + (d - r \sin \theta)^2 \right]^{\frac{1}{2}} \]
Real Loudspeaker System

Crossover: 1.2 kHz
4th order Linkwitz-Riley

LPF on one LF driver: 750 Hz
4th order Linkwitz-Riley

APF on other LF driver: 750 Hz
2nd order Butterworth
Real Loudspeaker System

Polar graph
1/3 octave, 800 Hz