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Loudspeaker Directivity Improvement Using Low Pass and All Pass Filters

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ABSTRACT

The response of loudspeaker systems employing multiple drivers within the same pass band is often less than ideal. This is due to the physical separation of the drivers and their lack of proper acoustical coupling within the higher frequency region of their use. The resultant comb filtering is sometimes addressed by applying a low pass filter to one or more of the drivers within the pass band. This can cause asymmetries in the directivity response of the loudspeaker system. A method is presented to greatly minimize these asymmetries through the use of low pass and all pass filters. This method is also applicable as a means to extend the directivity control of a loudspeaker system to lower frequencies.

1. INTRODUCTION

It is quite common for loudspeaker systems to utilize dual low frequency drivers and a high frequency device in a two-way design. This is often done to increase the low frequency output capability of the system in an attempt to match that of a horn loaded high frequency driver. It is also done, with the low frequency drivers placed symmetrically around the high frequency driver, to help the directivity response of the system remain symmetrical regardless of the types of crossover filters employed [1]. For the symmetrical case the low frequency drivers may either be placed horizontally on each side of the high frequency device or they may be placed vertically above and below the high frequency device. The former is typical of numerous line array modules and center channel home theater units while the latter can be seen in some free standing, bookshelf and studio monitoring loudspeaker systems.

The spacing of the woofers as dictated by their physical size usually results in comb filtering at some frequency within their pass band. Low pass and all pass filters will be applied to the drivers used in a hypothetical dual low frequency driver loudspeaker system to show how the comb filtering effects can be greatly minimized. Computer modeling is used to graphically illustrate the results.

The model utilizes simple omni-directional point sources to better detail the differences between an ideal case of uniform response and the summation of the modeled system, both on and off axis. By not considering the response of the individual transducers the analysis presented here can better focus on the variables of driver spacing/time offset and filter functions [2]. For the implementation of the methods presented here it is assumed that the transducers employed are reasonably uniform and well behaved in their response or can be made so through other means.

This decoupling of the transducer's response from the analysis will make the information presented applicable to the majority of loudspeakers in general which have similar transducer compliments and geometrical configurations. It will also aid in simplifying the interpretation of the graphical representation of directivity (directivity maps) by not confounding the reader with response anomalies of the individual drivers but only show the deviations from an ideal case of uniform response. The use of point sources in no way detracts from the validity or applicability of the model and the proposed method to actual devices.

Finally, the method presented is applied with slight modification to the measured data of a real loudspeaker system. The modification is desirable as a means to extend the directivity control of the system to a lower frequency region.

2. DRIVER SPACING & RESULTANT DIRECTIVITY

2.1. Modeling Method

The complex pressure of a point source is given by

$$\tilde{p}(\vec{r}) = \frac{\tilde{A}}{|\vec{r}|} \exp[\, j(\varphi - \vec{r} \cdot \vec{k})] \tag{1}$$

where $\vec{r}$ is the observation point, $\vec{k}$ is the wave vector and $\tilde{A}$ and φ are the angle and frequency dependent complex (magnitude and phase) correction for a given source. The combined pressure of all contributing sources may be calculated by the complex summation [3]

$$\tilde{p}_{sum}(\vec{r}) = \sum_i \tilde{p}_i(\vec{r}) \tag{2}$$

The point sources used to represent the individual drivers have identical output levels and are uniform with respect to frequency, phase and directivity. The summations are calculated at a frequency resolution of 1/24 octave and an angular resolution of 5°. The observation point is chosen sufficiently far from the sources such that the distance between the individual sources is small by comparison. This assures that our calculations are valid for the far field. The calculations were performed and most of the graphs generated by EASE SpeakerLab [4]. All response calculations and graphs are for the plane through the center line of the loudspeaker system and of all the individual drivers comprising the system. The calculated system response is normalized to the on-axis response. These results are then smoothed to 1/3 octave for display unless otherwise noted.

2.2. Directivity Map Graphical Display

The primary graphical display of directivity used will be the directivity map. The first instance of this type of map can be seen in Figure 3. The directivity map displays frequency (x-axis) and radiation angle (y-axis) for a given plane. The level of the system response at a particular frequency and radiation angle is denoted by the color of the graph. This gives much more information at a glance than the traditional polar response graph which can only display information about a single frequency or a single averaged bandwidth. When detailed information about level at a given frequency or averaged bandwidth is required the use of the polar response graph is appropriate.

The off-axis response displayed in the directivity map is normalized to the on-axis response. Therefore, the on-axis response always appears flat, 0 dB. A null in the on-axis response manifests itself as a peak in the off-axis response.

If a slice is taken horizontally through the directivity map at a given angle and the level graphed vs. frequency, the display would be that of a typical frequency response graph, normalized to the on-axis response. If a slice is taken vertically through the directivity map at a given frequency and the level graphed vs. the radiation angle, the display would be that of a typical polar response graph.

The directivity maps presented here are shown from 20 Hz to 20 kHz and for the front hemisphere from -90° (directly below the loudspeaker system) to +90° (directly above the loudspeaker system).

2.3. Example Loudspeaker System

An example loudspeaker system is shown in Figure 1 for a symmetrical arrangement of drivers. This illustrates the relative distance between drivers, d, and the radius, r, from a reference point on the loudspeaker system to the observation point. This is not drawn to scale.

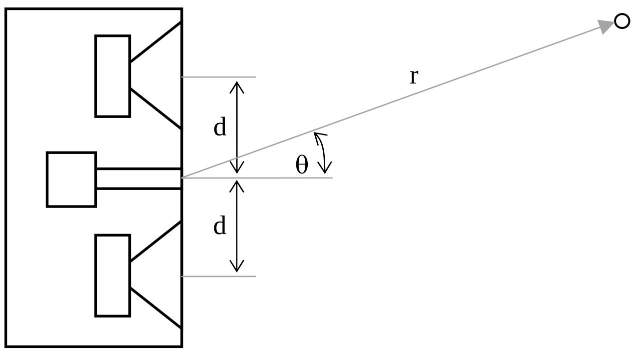

Figure 1: Geometry of example loudspeaker system

D'Appolito's analysis of the interaction between the two low frequency drivers and the high frequency driver shows that their symmetrical placement will yield symmetrical directivity lobing in the crossover region [1]. Subsequently, Konar [5] reveals that an examination of the interaction of the two low frequency drivers with each other, while symmetrical, can cause nulls in the off-axis response of the system. These nulls occur at progressively lower frequencies for increasing values of d and |θ|. These nulls may be problematic if they occur within the intended coverage angle of the loudspeaker system and at a lower frequency than the crossover to the high frequency driver. To minimize the detrimental effects of these nulls they should either occur at sufficiently large off-axis angles, outside of the intended coverage angles, or at sufficiently high frequency so that the output from the high frequency driver masks them. The lowest frequency at which off-axis nulls begin to occur within a specified forward radiation angle can be calculated using one of his equations shown here as Equation (3).

If our example system in Figure 1 uses 250 mm (10 in) low frequency drivers and a high frequency device with a 64 mm (2.5 in) wide exit, the spacing, d, should be approximately 170 mm (6.75 in). These values also conveniently equate to d = λ for a frequency of 2 kHz. The following graphs may be easily normalized to this frequency for any value of d. We will set the radius, r, to be 20 meters.

For the selected spacing Equation (3) can be used to determine the lowest frequency null within +/-90°, which is 500 Hz. This correlates very well with the polar response graph in Figure 2 and the directivity map in Figure 3. Both of these graphs are for the combined response of the two low frequency drivers. The high frequency driver is not active.

Clearly the first, lower frequency, nulls that are occurring at small off-axis angles are not desirable. For an included coverage angle of 90° (+/-45°) we can see that the first null occurs at approximately 700 Hz. This is typically too low of a frequency to have sufficient output from the high frequency driver to mask this null. The system directivity can be seen in Figure 4 when the high frequency driver is turned on and 1.2 kHz, 4th order Linkwitz-Riley low pass and high pass filters are used for the crossover.

$$f_x = \frac{1}{2} c * \left| \left\{ \left[(r\cos\theta)^2 + (d + r\sin\theta)^2 \right]^{\frac{1}{2}} - \left[(r\cos\theta)^2 + (d - r\sin\theta)^2 \right]^{\frac{1}{2}} \right\}^{-1} \right| \qquad (3)$$

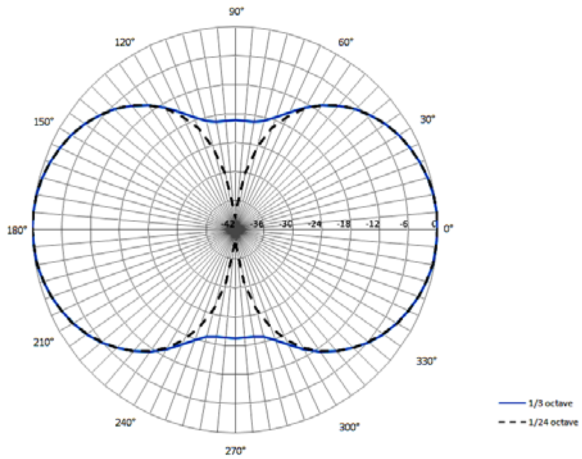

Figure 2: Polar graph of two LF drivers at 500 Hz; 1/3 oct. (–) and 1/24 oct. (--)

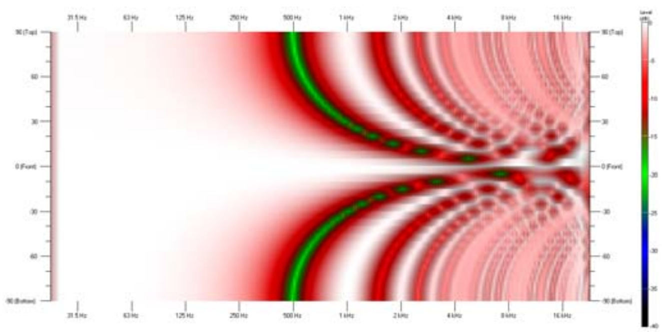

Figure 3: Directivity map of two LF drivers

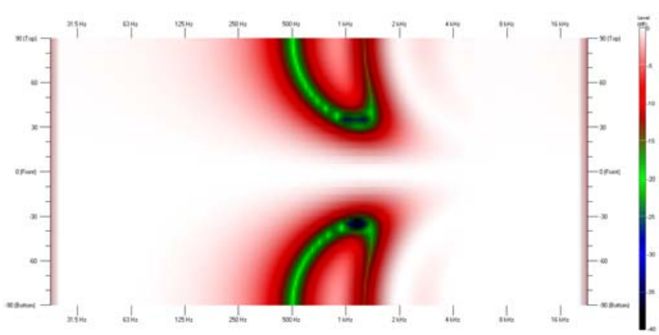

Figure 4: Directivity map of system with 1.2 kHz, 4th order Linkwitz-Riley crossover

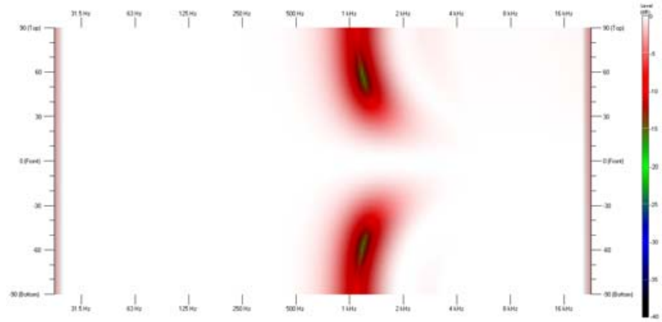

Figure 5: Directivity map of system response with one LF driver off

The desired system response would exhibit only the nulls due to the spacing between the low and high frequency drivers as well as the crossover filters used. For our example this would range from approximately -5 dB at 40° off-axis to -15 dB at 90° off-axis. This is shown in Figure 5 by turning off one of the low frequency drivers.

3. OPTIMIZING DIRECTIVITY RESPONSE

3.1. Minimizing Off-Axis Nulls

To eliminate or at least minimize the off-axis nulls that occur below the crossover frequency, the output of one of the low frequency drivers needs to be attenuated. This should take place between f_x, given by Equation (3) for the relevant included coverage angle and driver spacing, and the crossover frequency. This is easily accomplished by inserting an additional low pass filter in the signal path feeding one of the low frequency drivers. If this low pass filter is no greater than first order there will be no detrimental effects to the system response. However, the 6 dB/octave slope of a first order filter may not provide adequate reduction of the signal from one of the low frequency drivers to sufficiently minimize the off-axis nulls unless its corner frequency is set very low. This would seem to render this driver useless for approximately one decade below the crossover frequency in our example.

Equation (3) may be modified to assist in determining the appropriate parameters for the additional low pass filter to yield adequate attenuation. This equation gives the relationship between d, r and θ such that the path length difference between the two low frequency drivers is one-half wavelength and thereby the greatest cancellation. By substituting other values for the leading coefficient, frequencies with phase relationships other than 180° may be calculated.

The general relationship for any value of relative phase shift, φ, between the two low frequency drivers is given by

$$f_\varphi = \frac{\varphi}{360} * \frac{c}{|r_1 - r_2|} \tag{4}$$

where φ is in degrees and the distance from each low frequency driver to the observation point is

$$r_1 = \left[(r\cos\theta)^2 + (d + r\sin\theta)^2\right]^{\frac{1}{2}} \tag{5}$$

$$r_2 = \left[(r\cos\theta)^2 + (d - r\sin\theta)^2\right]^{\frac{1}{2}} \tag{6}$$

Equation (4) may be used to find the frequencies for which φ gives 120° and 90° phase shift between drivers at the specified off-axis angle. The frequency at which φ equals 120°, $f_{120°}$, is of particular interest. Below this frequency no cancellation will occur as there is only constructive interference between the two low frequency drivers. Above this frequency there will be cancellation due to destructive interference. If off-axis nulls are to be minimized, the level difference between the two low frequency drivers should increase above $f_{120°}$. The question now becomes how much level difference is required.

An expression relating the summation of the pressure from two sources with a defined phase difference is given in [6] as

$$P_T = \sqrt{P_1^2 + P_1^2 + 2P_1P_2\cos\varphi} \tag{7}$$

where P_T is the total pressure magnitude, P_1 and P_2 are the pressure magnitudes of the individual sources and φ is the difference of the pressure phase angles from the individual sources. On-axis the path length to both sources is the same, so φ is 0°. The ratio of the off-axis pressure at a position where φ is 180° to the on-axis pressure is given by

$$\frac{P_{T180°}}{P_{T0°}} = \frac{\sqrt{P_1^2 + P_2^2 - 2P_1P_2}}{\sqrt{P_1^2 + P_2^2 + 2P_1P_2}} \tag{8}$$

If we let *x* be the ratio of pressures P_1/P_2 then by substitution we have

$$\frac{P_{T180°}}{P_{T0°}} = \frac{\sqrt{P_1^2 + x^2P_1^2 - 2xP_1^2}}{\sqrt{P_1^2 + x^2P_1^2 + 2xP_1^2}} \tag{9}$$

We can then set P_1 to unity as a normalization factor to further simplify.

$$\frac{P_{T180°}}{P_{T0°}} = \frac{\sqrt{1 + x^2 - 2x}}{\sqrt{1 + x^2 + 2x}} = \frac{x-1}{x+1} \tag{10}$$

Finally, if we let $P_{R\ off\text{-}axis}$ represent the off-axis pressure ratio $P_{T180°}/P_{T0°}$ and solve for *x* we are left with

$$x = \frac{P_1}{P_2} = \frac{1 + P_{Roff-axis}}{1 - P_{Roff-axis}} \tag{11}$$

To limit the level of the off-axis response deviation to no more than -3 dB from the on-axis level requires that $P_{R\ off\text{-}axis} = 0.7071$. Using Equation (11) indicates that the ratio of pressures $P_1/P_2 = 5.848$. This is a level difference of 15.3 dB between the sources. This means that the level of one of the low frequency drivers must be reduced 15.3 dB at $f_{180°}$ compared to the other low frequency driver.

Limiting the off-axis deviation to no more than -1 dB means that $P_{R\ off\text{-}axis} = 0.8913$. This results in a level reduction requirement of 24.8 dB at $f_{180°}$ between the two drivers. We shall adopt this 1 dB deviation as our criterion for minimizing the off-axis nulls. The interval from $f_{90°}$ to $f_{180°}$ is one octave. By employing a 4th order low pass filter (24 dB/octave) with a cutoff frequency of approximately $f_{90°}$ this criterion can be met.

For our example loudspeaker system Equation (4) tells us that $f_{90°}$ = 250 Hz for θ = 90°. In addition to the 1.2 kHz crossover filters, we will apply a 250 Hz, 4th order Linkwitz-Riley low pass filter to one of the low frequency drivers. This results in the directivity response shown in Figure 6. Here we can see that the off-axis nulls that had been present in the 300 – 800 Hz region (Figure 4) have been greatly minimized.

The use of this low pass filter has also brought unwelcomed side effects. There is now asymmetrical lobing in the low frequency region of the directivity response (Figure 6 and Figure 7), as well as a large null (dip) in the on-axis response (Figure 8). This is due to the phase shift of the added low pass filter exceeding 120° prior to its output level being sufficiently reduced.

The 6 dB level increase in the low frequency region, below 100 Hz, is due to the equal level contributions from the two low frequency drivers.

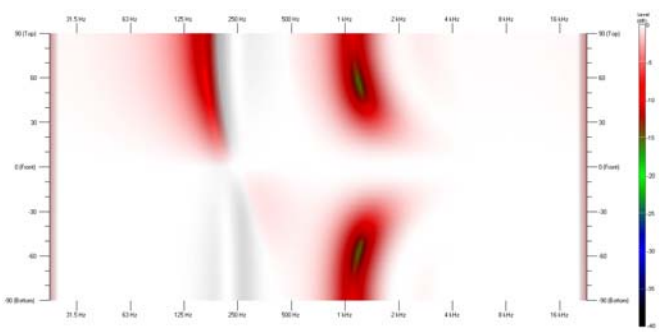

Figure 6: Directivity map of system with crossover and 250 Hz, 4th order Linkwitz-Riley LP filter on one LF driver

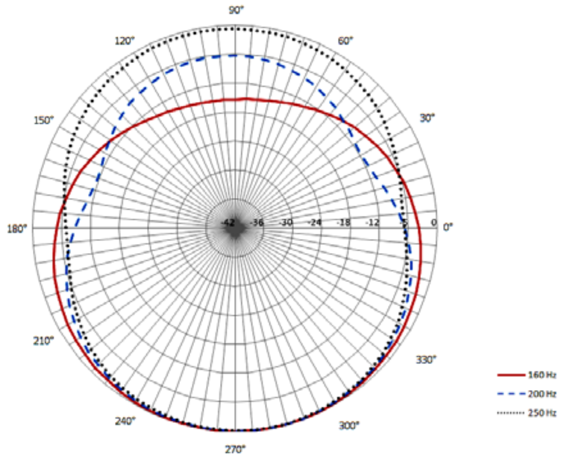

Figure 7: Polar graph of system with crossover and 250 Hz, 4th order Linkwitz-Riley LP filter applied to one LF driver; 160 Hz (–), 200 Hz (--) and 250 Hz (..)

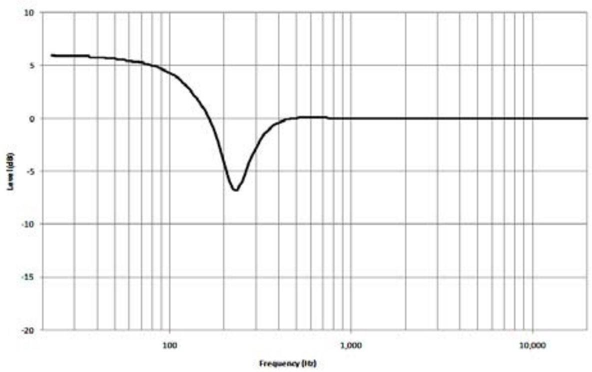

Figure 8: On-axis response of system with crossover and 250 Hz, 4th order Linkwitz-Riley LP filter applied to one LF driver

3.2. Phase Compensation for Other Drivers

To eliminate the cancellations, both on and off axis, caused by the phase shift of the additional low pass filter that was applied to the first low frequency driver, additional phase shift needs to be introduced to the second low frequency driver. An all pass filter with phase shift identical to that of the added low pass filter can be used to accomplish this. Once this all pass filter has been implemented, the combined response of the low frequency drivers suffers no ill effects other than the additional phase shift of the filters.

It should be noted that the responses of the second low frequency driver and the high frequency driver are no longer in-phase at all frequencies through the crossover region due to the additional phase shift of the all pass filter. This will give rise to asymmetrical lobing of the directivity response [7]. The effects of this on the directivity response of the system will be similar to those caused by the phase shift of the added low pass filter applied to the first low frequency driver when combined with the non-phase compensated output of the second low frequency driver. The only difference being that these effects will occur at frequencies within the crossover region. This is similar to the issues documented by D’Appolito in [8]. The application of his binary-tree topology (Figure 9) is an effective solution to this problem.

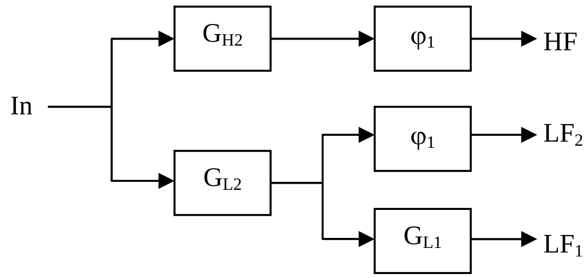

Figure 9: Binary-tree topology of signal flow

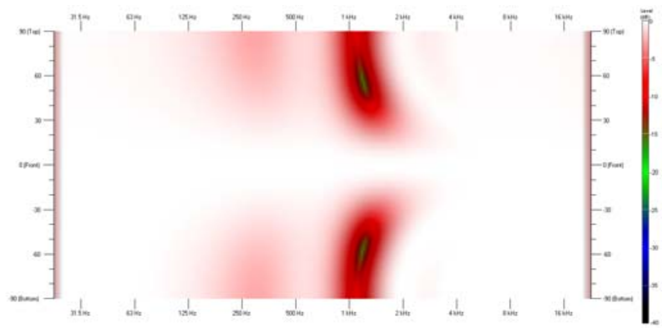

Figure 10: Directivity map of system with filter parameters from Table 1

Here we see that an all pass filter, with phase response φ_1 (which has the same phase response as the additional low pass filter G_{L1}) is added not only to the second low frequency driver but also to the high frequency driver. This additional phase shift applied to the high frequency driver now assures that the combined response of all filter sections used for the low pass and the combined response of all filter sections used for the high pass yield an in-phase response through the crossover region for LF_2 and HF. This is true so long as filter sections G_{L2} and G_{H2} have an in-phase response (4th order Linkwitz-Riley filters as an example).

3.3. Final System Response

The system directivity response can now be calculated with the filtering applied as shown in the signal flow diagram of Figure 9. The same filter parameters are used for this as in the previous calculations and are detailed in Table 1. The system directivity map with these filters is shown in Figure 10.

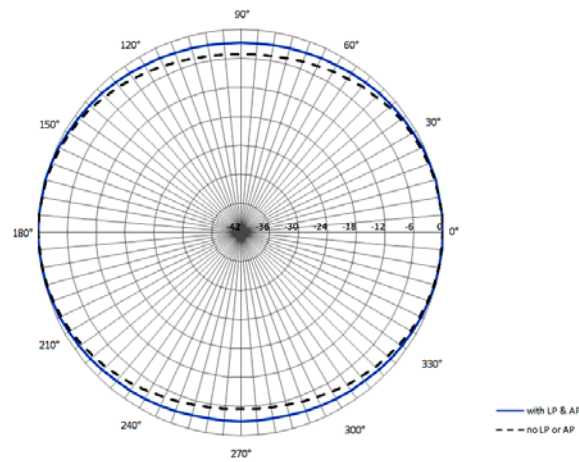

Figure 11: Polar graph of system for 315 Hz one-third octave band showing response with additional LP and AP filters (–) and without (--)

Filter	Alignment	Order	Frequency
G_{H2}	Linkwitz-Riley	4	1.2 kHz
G_{L2}	Linkwitz-Riley	4	1.2 kHz
G_{L1}	Linkwitz-Riley	4	250 Hz
ϕ_1	Butterworth	2	250 Hz

Table 1: Filter parameters

There are still some minor level reductions at large off-axis angles in the vicinity of 300 Hz relative to the on-axis response. Inspection of the polar response graph (Figure 11) in this frequency region shows that these are less than 3 dB at the extremes of +/-90°. The additional low pass and all pass filters have increased the level of the combined response at these extremes by approximately 2 dB compared to the original system response without the additional filtering.

The only major off-axis nulls remaining are in the crossover region between the low frequency and high frequency drivers. This is very similar to the response with one of the low frequency drivers muted shown in

Figure 5. To see this more clearly the polar graph in Figure 12 compares the response in the crossover region for the three different system configurations; 1) with only one low frequency driver active, 2) both low frequency drivers active but without the added low pass/all pass filters and 3) two low frequency drivers with the added low pass/all pass filters. The response with the additional filtering overlays the response of the system using just one active woofer. This graph also shows that the off-axis nulls of almost -30 dB at 35° have been greatly minimized, to approximately -7 dB, due to the added filtering.

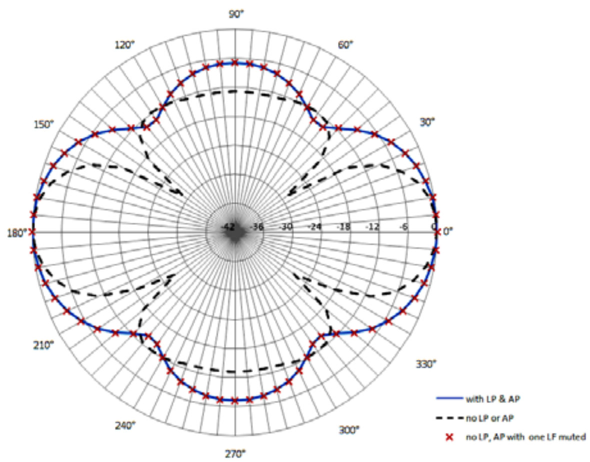

Figure 12: Polar graph of system of 1.25 kHz one-third octave band showing response with additional LP and AP filters (–), without filters (--) and with one LF driver muted (x)

The on-axis magnitude response with and without the added filtering is shown in Figure 13. The only difference seen here is the frequency region in which the system has greater output capability due to the output of two low frequency drivers. At first glance the response without the added filtering might be viewed as being more desirable than the response with the low pass/all pass filtering. However, it must be remembered that this response only occurs on-axis. As one moves off-axis the response of the system without the added filtering changes greatly and should be avoided.

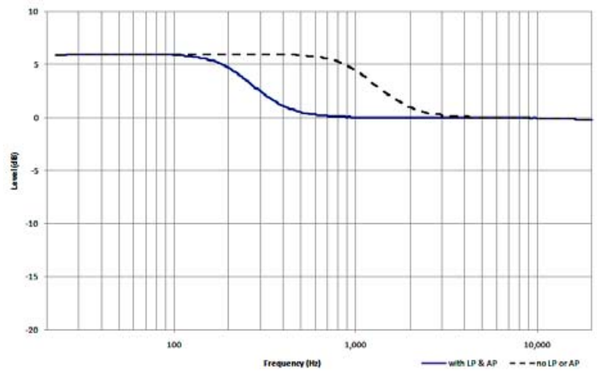

Figure 13: On-axis magnitude response of system with additional LP and AP filters (–) and without filters (--)

3.4. Detrimental Effects

There is a price to be paid for the more consistent directivity response gained via this method. There is one additional rotation in the phase response of the system caused by the additional filtering (Figure 14). This will also affect the impulse response of the loudspeaker system (Figure 15). For applications requiring uniformity of coverage this would seem to be a small price to pay. For other applications where the on-axis transient response is paramount and the off-axis is secondary this may not be an acceptable compromise.

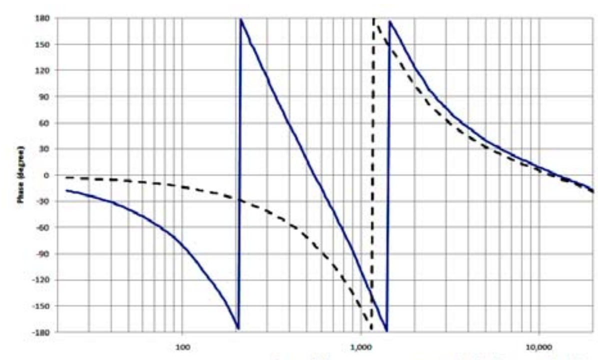

Figure 14: On-axis phase response of system with additional LP and AP filters (–) and without filters (--)

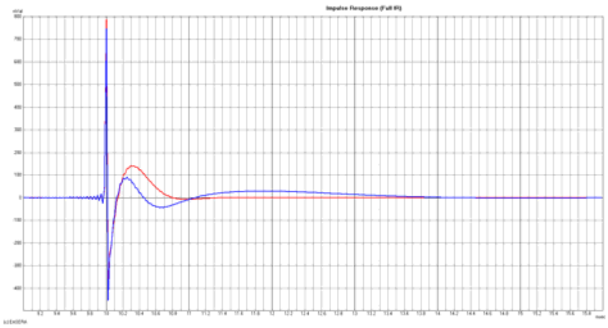

Figure 15: On-axis impulse response of system with additional LP and AP filters (–) and without filters (–)

4. OBSERVATIONS FOR APPLICATIONS WITH REAL SOURCES

Real acoustical sources used for loudspeaker systems typically are not omni-directional as are the point sources used for the analysis presented here. They will have some inherent directivity. To keep the directivity response of the system as consistent as possible with respect to frequency a particular spacing of low frequency drivers may be desired. This should be based on the directivity of these drivers, the directivity of the higher adjacent pass band and the crossover frequency region to that pass band. Careful spacing and choice of the frequency for the low pass/all pass filtering can help extend the directivity control of the system to frequencies below that of the crossover for a system using a high frequency horn with well behaved directivity.

For this to occur, it is desirable for the angles at which the response of the high frequency horn is -6 dB relative to the on-axis level also be matched by the low frequency drivers. If no additional low pass/all pass filtering were used this would occur at $f_{120°}$. This is because on-axis the drivers are in phase and sum to +6 dB compared to a single driver. With a phase difference of 120° the summation of the two drivers is 0 dB relative to a single driver. Clearly a frequency higher than $f_{120°}$ must be used as the corner frequency for the low pass/all pass filters.

Equation (11) can be used to determine the higher corner frequency at which the low pass/all pass filters should be set. For an off-axis level difference of -6 dB, $P_{R\ \text{off-axis}}$ = 0.5012. This results in a required level difference between the two low frequency drivers of 9.6 dB at $f_{180°}$.

A 4th order Linkwitz-Riley low pass filter will have an attenuation of 9.6 dB at a frequency approximately 1.19 times its corner frequency, f_c. By setting the filter f_c lower by a reciprocal amount, 0.84, the output level of the filter at the original f_c will now be -9.6 dB instead of -6 dB. This can be a bit too high and allow more narrowing than is desirable at higher frequencies within the pass band of the low frequency drivers. Refer to Equation (3) and Figure 3 to see how the nulls occur at smaller off-axis angles and higher frequencies.

The f_c calculated by Equation (11) and the appropriate scaling factor of the filter alignment and order used is a good starting point but it may need to be lowered. How much it will need to be lowered will depend on how much higher in frequency the low frequency drivers will need to work in order to reach the crossover frequency to the high frequency driver. If the spacing of the low frequency drivers has been chosen well for the intended crossover frequency this should be a minimal amount, hopefully no more than 1/3 to 1/2 of an octave. For the spacing and crossover frequency in our example we will lower by an additional 1/6 octave.

An example of the application of this method using a real loudspeaker is shown in Figure 16. The filters are applied to the directivity balloons of each individual source. The drivers, spacing and the crossover filters used here are the same as in the previous example with point sources. The difference is that the frequency of the additional low pass/all pass filters is increased from 250 Hz to 750 Hz. The nominal coverage angle of the high frequency horn is 60° (+/-30°). For Equation (4) setting θ = 30° and φ = 180° yields $f_{180°}$ = 1,000 Hz. Scaling this by 0.84 gives a frequency of 840 Hz. Reducing this further by 1/6 octave, a factor of 1.122, gives an f_c of 750 Hz for the additional low pass/all pass filtering.

Setting the low pass/all pass filters to 750 Hz allows for some cancellations between the two low frequency drivers to occur. This is beneficial for the loudspeaker system because the desired directivity, or coverage angle, exhibited by the high frequency horn is achieved to a frequency well over an octave below the crossover frequency of 1.2 kHz between the low frequency driver and the high frequency horn. This is further illustrated in a comparison of the polar response at 800 Hz to that of the same system with the low pass/all pass filtering set at the original 250 Hz and also with no additional low pass/all pass filtering (Figure 17). It should be clear from looking at this graph that the beamwidth, or included angle, of the front lobe in this frequency region can be adjusted between the limits of no low pass/all pass filtering and low pass/all pass with the original 250 Hz corner frequency by simply changing the corner frequency of the low pass/all pass filters.

Unfortunately for this particular loudspeaker the directivity just above crossover (1.2 – 2.0 kHz) is not maintained well. This is caused by the high frequency horn losing pattern control due to its small physical size in this plane.

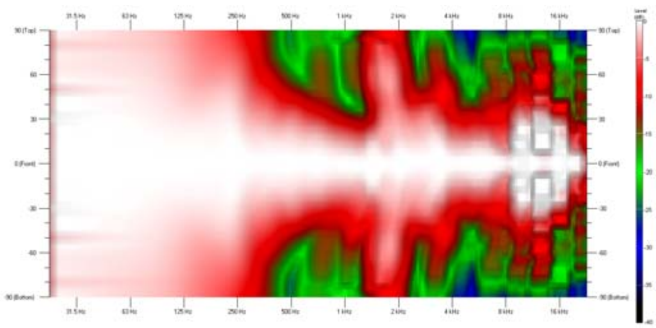

Figure 16: Directivity map of a real loudspeaker system response with measured data of the individual acoustical sources

5. CONCLUSIONS

We have reviewed the off-axis interference effects that occur when using physically separated sound sources. We have also proposed a criterion to greatly minimize these effects at all off-axis angles and detailed how this can be accomplished using additional low pass and all pass filters. An example loudspeaker system using point sources and computer-aided modeling graphically demonstrated the effectiveness of this technique. A real loudspeaker system showed how changing the frequency of the additional low pass and all pass filters can help extend the directivity control to well below the crossover region.

The implementation of these methods should be readily available to many practitioners given the proliferation of relatively inexpensive DSP devices.

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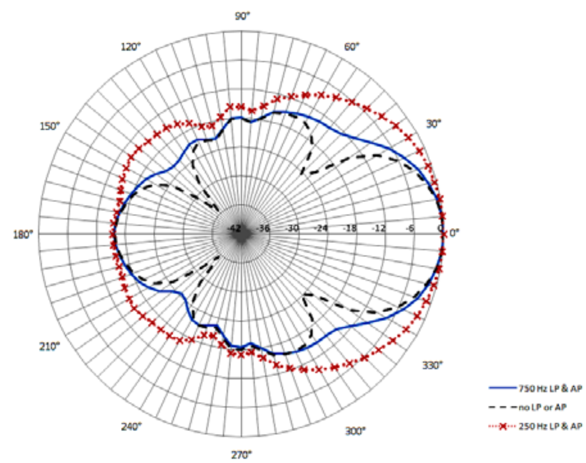

Figure 17: Polar graph of a real loudspeaker system for 800 Hz one-third octave band showing response with 750 Hz LP and AP filters (–), without filters (--) and with 250 Hz LP and AP filters (x)

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