The Quadratic-Throat Waveguide®:

A White Paper On An Invention
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A Brief History of Horn Types:

**Conical horns:** Known generally as straight-sided horns, conical horns are the oldest and simplest design. A cone describes their shape, and being a section of a perfect sphere of sound radiation, there is no phase or amplitude distortion introduced to the spherical-segment wave-front as it radiates out from the origin.

The simple formula for a conical horn is:

\[ S = S_1 x^2 \]

- \( S \) = the area at the horn mouth
- \( S_1 \) = the area at the horn throat
- \( x \) = the length of the horn

Variations using rectangular cross-sections and flat planar walls provided directional control at the expense of some wave-front distortion. Some of these variations no longer follow the strict definition of a conical horn since the expansion rate of the horn area is no longer a conical expression mathematically. The weakness of conical horns lies in their acoustical loading characteristics for the transducer, which is insufficient at the low-frequency end of its desired frequency range.

**Exponential Horns:** Exponential horns are those where the horn length is exponentially related to the horn area. The expression for an exponential horn is:

\[ S = S_1 e^{mx} \]

- \( S \) = the area at the horn mouth
- \( S_1 \) = the area at the horn throat
- \( m \) = the flare constant
- \( x \) = the length of the horn
Once in the proper bandpass region for a given size, an exponential horn presents a fairly consistent acoustical load to its driver. This helps both output level and evenness of frequency response, and is what makes horn designers incorporate this particular horn type into many popular compound horn configurations today. However, the exponential flare causes high frequency beaming, and this is the downfall of the exponential horn.

**Multicell Horns:** Multicell horns are simply a group of symmetrical, narrow-dispersion, exponential horns assembled into an array. This approach is better than the high-frequency beaming problems of single, large exponential horns in that their high-frequency loss due to beaming is restricted to much smaller seating segments within the arrays coverage area. However, they exhibited midrange beaming in both the horizontal and vertical directions and are very expensive to fabricate.

**Radial Horns:** Radial horns are a variation on exponential horns. Take the cross-sectional view of an exponential horn flare and rotate it as a radius of a circle about an axis of the circle, the resulting shape is a radial horn flare.

The straight sidewalls provide good pattern control across most of the radial horn's passband, but the exponential vertical component still results in vertical beaming of the high frequencies. Altec “Sectoral” horns are popular examples of radial horns.
**Constant Directivity Horns:** The term “Constant Directivity” is a trademark of Electro-Voice but has become somewhat of a catchall phrase to describe constant-beamwidth horns. In 1975, Electro-Voice introduced a single-cell horn that consisted of three-stages. The design incorporates a hybridized hyperbolic/exponential throat section coupled to a conical, vertically flared, radial bell section. Flanges that correct for mid-range beaming caused by edge diffraction are comprised of a second, wider conical, vertically flared, radial bell-section. As with classic radial horn designs, the sidewalls are straight, but in two flange sections.

Having constant beamwidth in both the vertical and horizontal directions, and an unprecedentedly high directivity index, or Q-factor, these horns became the model for virtually all new horn designs for the next decade. Additionally, the horn loaded the driver well and as a result, sounded very good.

The weakness of this design is in the fact that the aspect ratio of the mouth appearance is not very uniform. By design, the narrower vertical angle has a horn dimension much smaller than the wider horizontal angle’s horn dimension. The greater dimension of the horizontal dispersion angle is also far more than the longest wavelengths produced by the driver required. Yet this over-designed horizontal horn size cannot be reduced because it would restrict the vertical dimension of the horn. This, in turn, would force vertical beamwidth control to too high a frequency limit to be acceptable. So the horn has to remain wider than necessary for the needed horizontal directivity control and has a less-than optimum vertical directivity control.
**Mantaray® Horns:** The Mantaray® horns by Altec, sought to improve the vertical directivity control over the EV design and provide a more symmetrical horn-mouth aspect ratio in the process. Rotating the horn 90 degrees, so that the horizontal expansion slot of the EV design is vertically oriented on the Altec design, does this. Since the coverage angle in the vertical is narrower than the horizontal, the expansion break has to be pushed forward producing a diffraction slot at the end of a long narrow tail of vertical exponential expansion. The vertical diffraction slot fills the horn bell, which is a two-stage design similar to the EV horn, but comprised of planar sections rather than radial sections.

The planar sections do not provide quite as good a driver-loading characteristic as the EV's radial sections, but they are adequate for an 800 Hz crossover frequency, and provide superior vertical control with a more symmetrical mouth size.

However, this approach does introduce a new problem for horns used in arrays. The term “apparent apex” was coined to describe the angular focal point of dispersion, where the coverage angles converge in each plane. The apparent apices do not occur at the same point on the axis of this particular horn design. This causes the curvature of the wavefront as it emanates from the horn to be ellipsoidal, rather than spherical and is, therefore, astigmatic. Arrayed horns would have a horizontal axis at the diffraction slot, while vertical arrays would have the axis at the horn mouth. This means that the horns can only be arrayed properly in one plane, and will necessarily be improperly arrayed in the other plane. In addition, the abrupt breaks in the flare rate of the horn at the junction of the bell sections have diffraction, reflection, and distortion components.
**Bi-Radial® Horns:** JBL refined the constant beamwidth concept one step further with the Bi-Radial® horn. Noting the abrupt breaks in the flare-rate via the sectioned bell flanges of the Altec design, JBL incorporates a radial design into both the horizontal and vertical expansions. This spawns the name Bi-Radial®. The horn still maintains the good loading and directional control via the diffraction slot and nearly straight first section of the bell like the Mantaray® horn. However, it also reduces the diffraction and reflective problems of joining two distinct planar flange sections by substituting the radial curve for the primary and secondary flares of the bell.

Since the design still incorporates the diffraction slot, and still has the differing apparent apices, the design still suffers from the same astigmatism as the Mantaray® horn.
The Problems With Most Current Constant-Beamwidth Horn Designs:

Most current popular horn designs share a common set of problems:

1. The ellipsoidal wavefront astigmatism caused by different horizontal and vertical apparent apices, which, by definition, is a characteristic of slot-loaded horn bells
2. Distortion at high sound-pressure levels caused by the slowing expanding exponential horn “tail”
3. Resonances caused by the parallel or near-parallel walls of the slowing expanding exponential horn “tail”
4. Reflections and diffraction effects caused by the angular breaks between multi-flanged sections of the horn, including the diffraction slot, primary and secondary flanges

Horns versus Waveguides:

Originally, a horn was generally used to increase the acoustic output of a transducer by providing improved loading as compared to directly coupling the transducer to the open environment. As stated by Harry Olsen in *Acoustical Engineering*, “The principal virtue of a horn resides in the possibility of presenting practically any value of acoustical impedance to the sound generator.” As a side benefit, it provided some directivity control to the dispersion of acoustical energy.

Today’s so-called “waveguides” are horns with the directional characteristics being the prime design criteria over optimum driver loading. This is due to the fact that today’s compression drivers are very efficient and have a much higher power-handling capability than in past decades. This allows the designer to pay more attention to the directional characteristics than the loading of the driver in newer horn designs.

The New Peavey Quadratic-Throat Waveguide®:

The Design Concept: Initially, a conical bell with straight sidewalls was chosen for two reasons. First, modern compression-driver performance is not as restricted by loading as in the past. Second, conical horns exhibit a spatially non-distorted spherical-segment wavefront characteristic.
The first challenge of this new design is to solve the astigmatism of the ellipsoidal wavefront caused by different horizontal and vertical apparent apices. This design moves the forward (usually horizontal) apparent apex back to the same point as the rear (usually vertical) one at the center of the throat entrance.

Now the challenge is to mate the necessary throat diameter at its entrance to the straight-sided horn walls and still meet the required design coverage angles. A circular arc performs this function quite well. The arc is formed between a point on the perimeter of the throat to a point where it is congruent with the angle of the sidewall. The axis of this arc is at the intersection of the two lines that define the radius of the arc. One line is drawn perpendicular to the axis of the horn at the throat entrance perimeter and the other is perpendicular to the point where the arc intersects the sidewall. Using this axis, an arc is formed that is tangent to the sidewall at one end and parallels the horn axis at a point on the throat entrance wall on the other. The design dictates that the angle of the arc is equal to one-half the dispersion angle of the horn in that plane.
This creates a horn that smoothly progresses from a circular to a rectangular cross-section without any abrupt changes in flare-rate. The circular shape at the throat evolves into a rectangle by allowing elliptical fillets to form where the corners of the final rectangle will be located. In this way a very smooth transition that enables the radiating wavefront to remain at right angles with the horn walls is maintained along the entire horn’s axis. The dimension that defines the wider angle evolves the straight sidewall nearer to the throat than the narrower-angle dimension.

It might be noted that the design still lacks one important feature that the earlier designs have, namely the secondary flanges that prevent mid-range beaming due to edge-diffraction effects. In order to keep the mouth size to a minimum, and to maintain the superior non-distorting conical expansion, a different solution was devised to avoid the addition of secondary flanges. Many types of acoustical foam were tried until a suitable material was found to line the edges of the horn mouth. This new foam edge successfully minimizes the diffractional effects that cause mid-range beaming with only slightly increasing the size of the horn mouth. Thus, this new design provides a very compact, very low distortion, constant-beamwidth horn with no astigmatism to warp the spherical-segment wavefront.
The mathematics that describes this design are not within the scope of this paper. However, as a point of information, the formula describing the cross-sectional area expansion rate of the throat section is in the form of a quadratic equation, hence the name Quadratic Throat. Its formula is as follows:

\[ S = Ax^2 + B\sqrt{r^2 - x^2} + C \]

If desired, the mathematics describing this horn design can be studied at length by referring to the AES paper by Charles E. Hughes. It is listed in the reference section at the end of this white paper.
Comparative Performance: The new design was tested versus standard constant-coverage horns using the same driver, throat entrance size for both design. The Quadratic-Throat Waveguide exhibited slightly more output level in the 600 to 900Hz region, the 1500 to 2200 Hz region, and the 7000 to 10,000 Hz region.

![Amplitude Response](image1)

**FREQUENCY RESPONSE COMPARISON**
(higher level is superior performance)

- - - - - - Standard constant-coverage horn
-------- Quadratic-Throat Waveguide®

For beamwidth control in the horizontal plane, it was superior in the 500-Hz region, but virtually identical for the rest of the bandpass.

![Horizontal Beamwidth](image2)

**HORIZONTAL BEAMWIDTH COMPARISON**
(closest to the ideal 90 degrees is superior performance)

- - - - - - Standard constant-coverage horn
-------- Quadratic-Throat Waveguide®
In the vertical beamwidth control plane, it was better across the entire bandwidth with the exception of the 1000 Hz region and the 2000 Hz region, where it performed equally.

**VERTICAL BEAMWIDTH COMPARISON**

(closest to the ideal 40 degrees is superior performance)

- - - - - - - Standard constant-coverage horn

- - - - - Quadratic-Throat Waveguide®

Where the Quadratic-Throat Waveguide® really outperformed the standard constant-beamwidth coverage horn was in harmonic distortion. At several power levels and across all frequencies, the 2nd harmonic distortion level was almost uniformly 3 to 4 dB better.

**SECOND-HARMONIC DISTORTION LEVEL COMPARISON**

(lower level of distortion is superior performance)

- - - - - - - Standard constant-coverage horn

- - - - - Quadratic-Throat Waveguide®
And finally, the 3rd harmonic distortion, the most irritating for the listener, averaged about 9 dB better than the conventional design. This is an easily discernable sound quality difference over the older designs.

![3rd Harmonic - 20W](image)

**THIRD-HARMONIC DISTORTION LEVEL COMPARISON**

(lower level of distortion is superior performance)

--- Standard constant-coverage horn
——— Quadratic-Throat Waveguide®

**Conclusions:** It has been demonstrated that a new generation of horns has been designed that clearly outperform the current industry standard offerings. The Quadratic-Throat Waveguide® advantages are as follows:

1. Because it has no diffraction slot, the Quadratic-Throat Waveguide® exhibits no horizontal vs. vertical astigmatism causing wavefront distortion and arraying problems.

2. It has no slowly expanding exponential “tail” section to cause high-SPL distortion or resonances due to its near parallel walls.

3. It has a smaller mouth area and resulting frontal size due to the use of foam edging rather than a secondary flange to minimize edge diffraction effects that cause mid-range beaming.

4. It produces less harmonic distortion due to the smooth waveform propagation through the quadratic throat and conical bell vs. a standard diffraction slot and secondary flange expansion breaks.

So, as a more compact horn, with lower spatial distortion to enable better arraying, and with far less audible harmonic distortion, the Peavey Quadratic-Throat Waveguide® represents the latest in new horn design technology in our industry. Peavey Electronics Corporation will initially be offering the Quadratic-Throat Waveguide® in its new ILS Speaker Series, available only to Peavey Architectural Acoustics Dealers. Applicable patents are pending.


4. Don B. Keele, Jr., “What’s So Sacred About Exponential Horns?” Audio Engineering Society, Pre-Print No. 1038 (F-3), (1975 May)


6. Charles E. Hughes, “A Generalized Horn Design to Optimize Directivity Control & Wavefront Curvature,” Audio Engineering Society, Pre-Print No. 5016 (F-6), (1999 Sept.)

7. Don B. Keele, Jr., telephone conversation, (2000 Mar.)

8. Charles E. Hughes, telephone conversation, (200 Mar.)